

# Linearity of the BOLD Response

*Peter A. Bandettini*

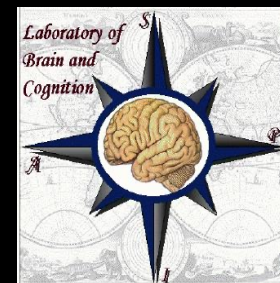
*Rasmus Birn*

*Ziad Saad*

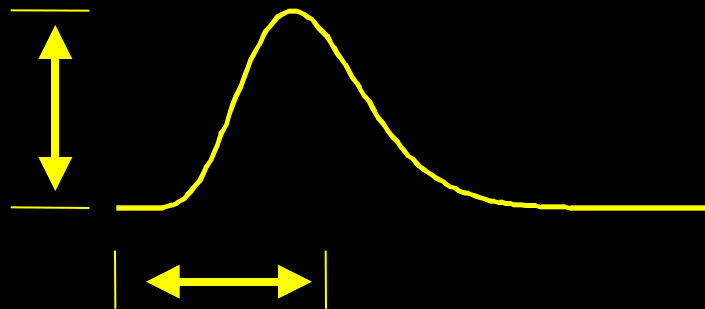
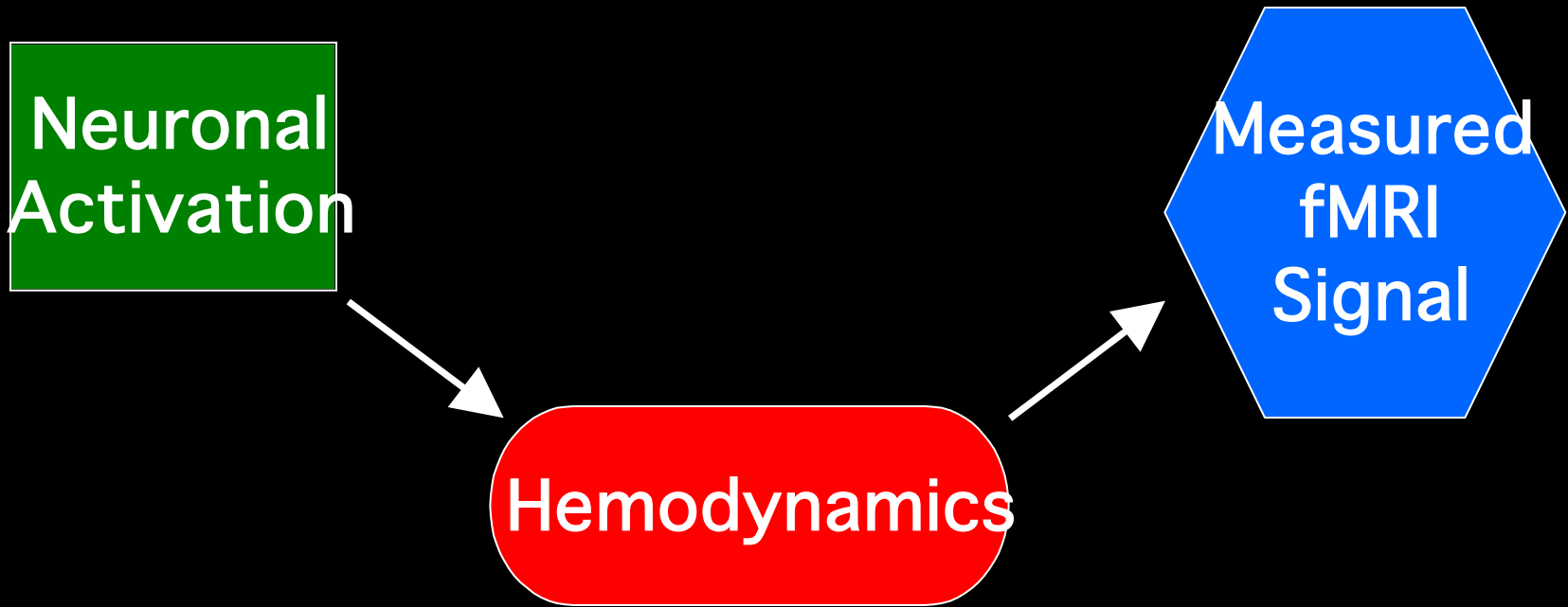
*Dan Kelley*



*Unit on Functional Imaging Methods  
Laboratory of Brain and Cognition,  
National Institute of Mental Health, NIH*

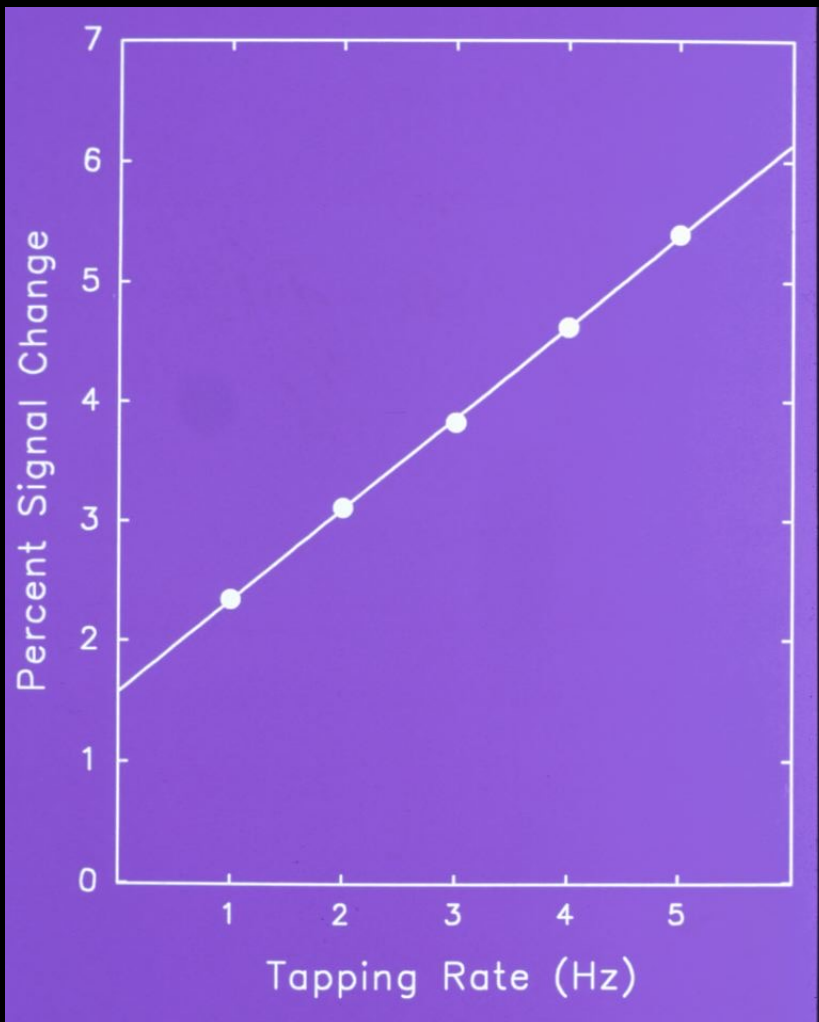


# Hemodynamic Transfer Function

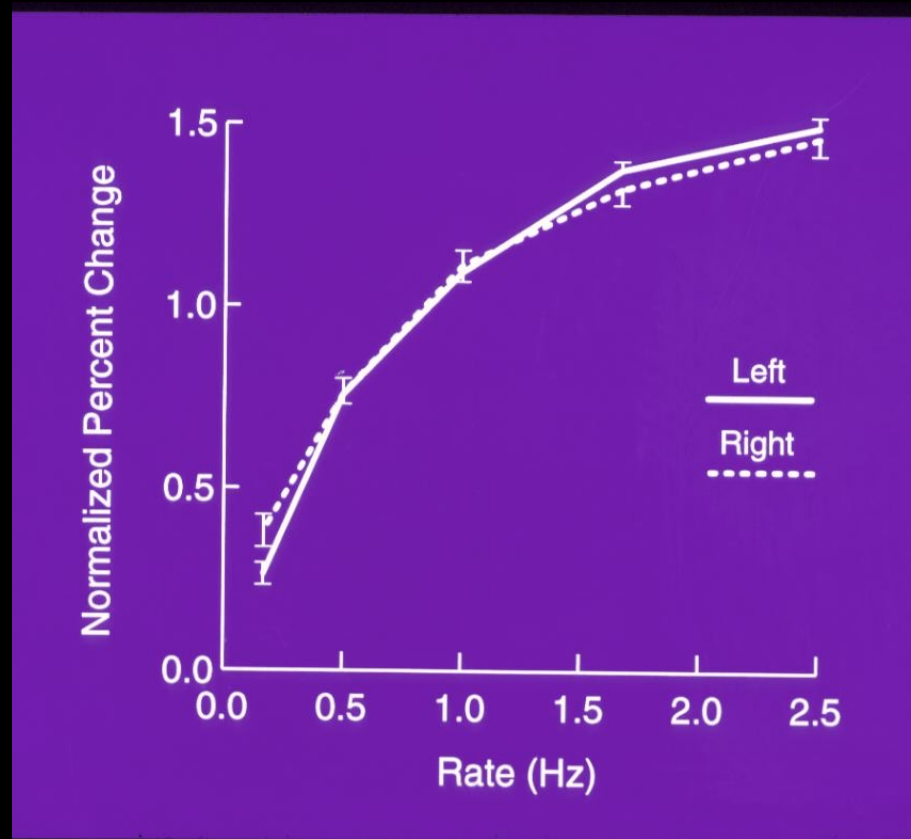


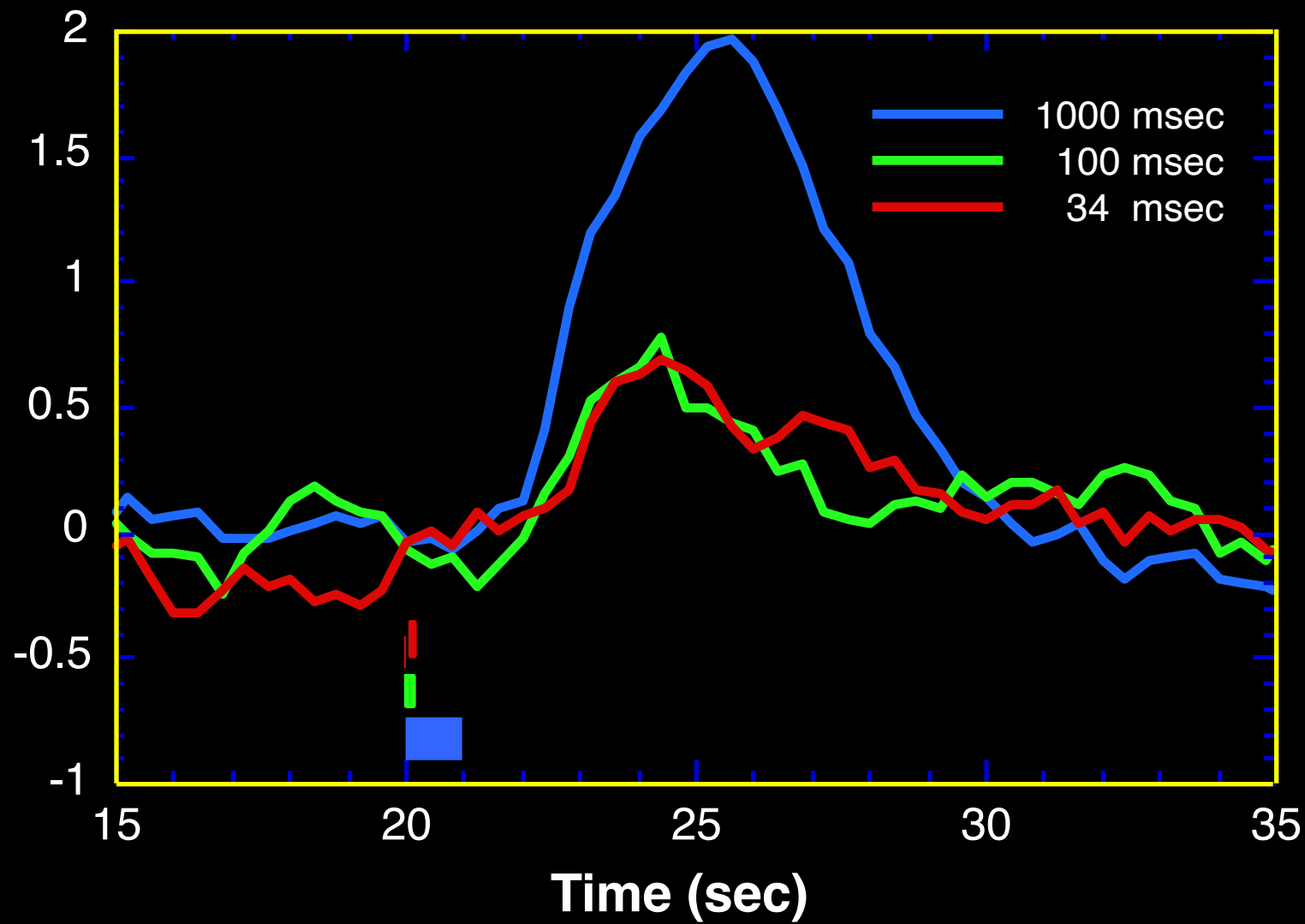
Physiologic Factors

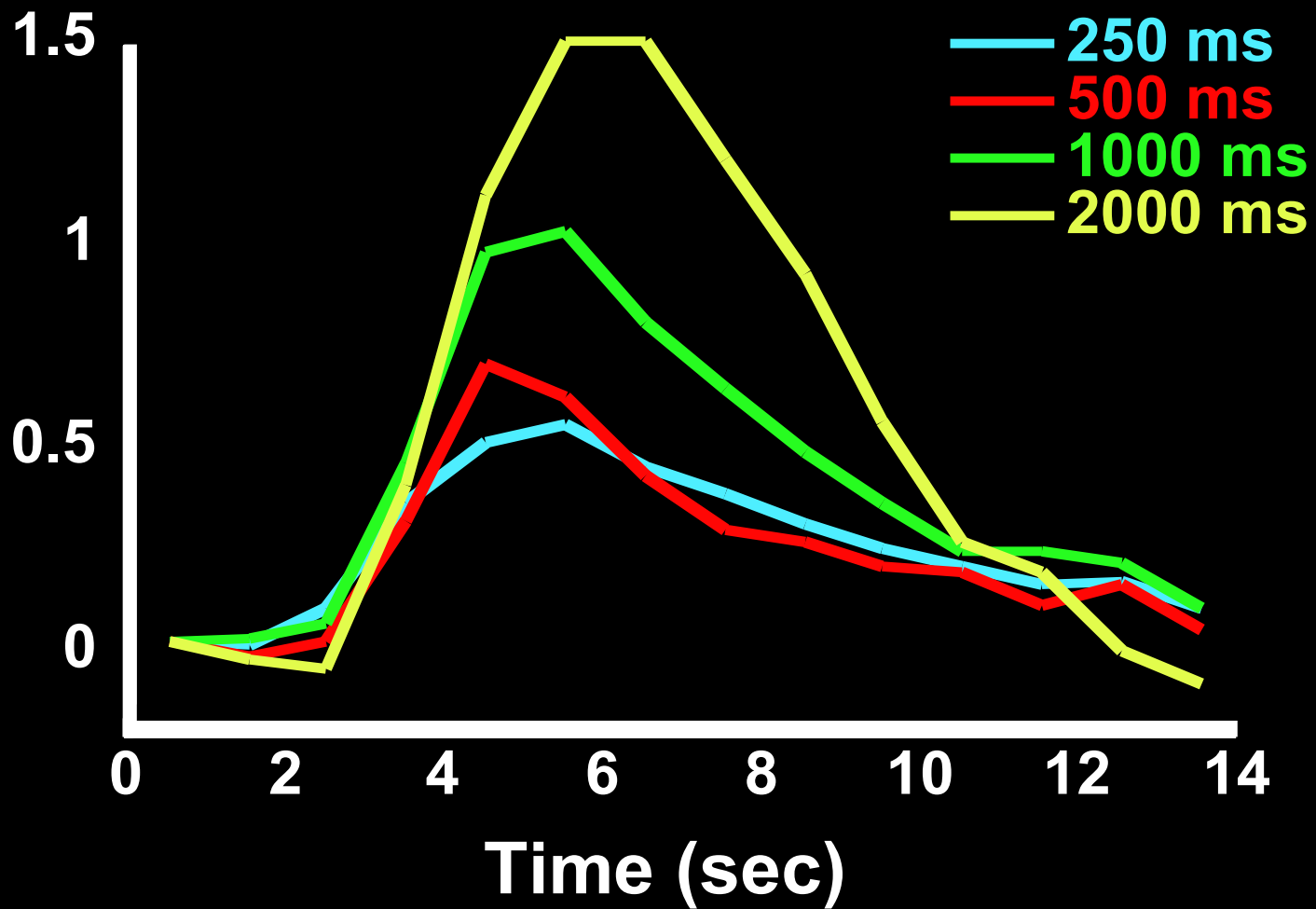
# Motor Cortex



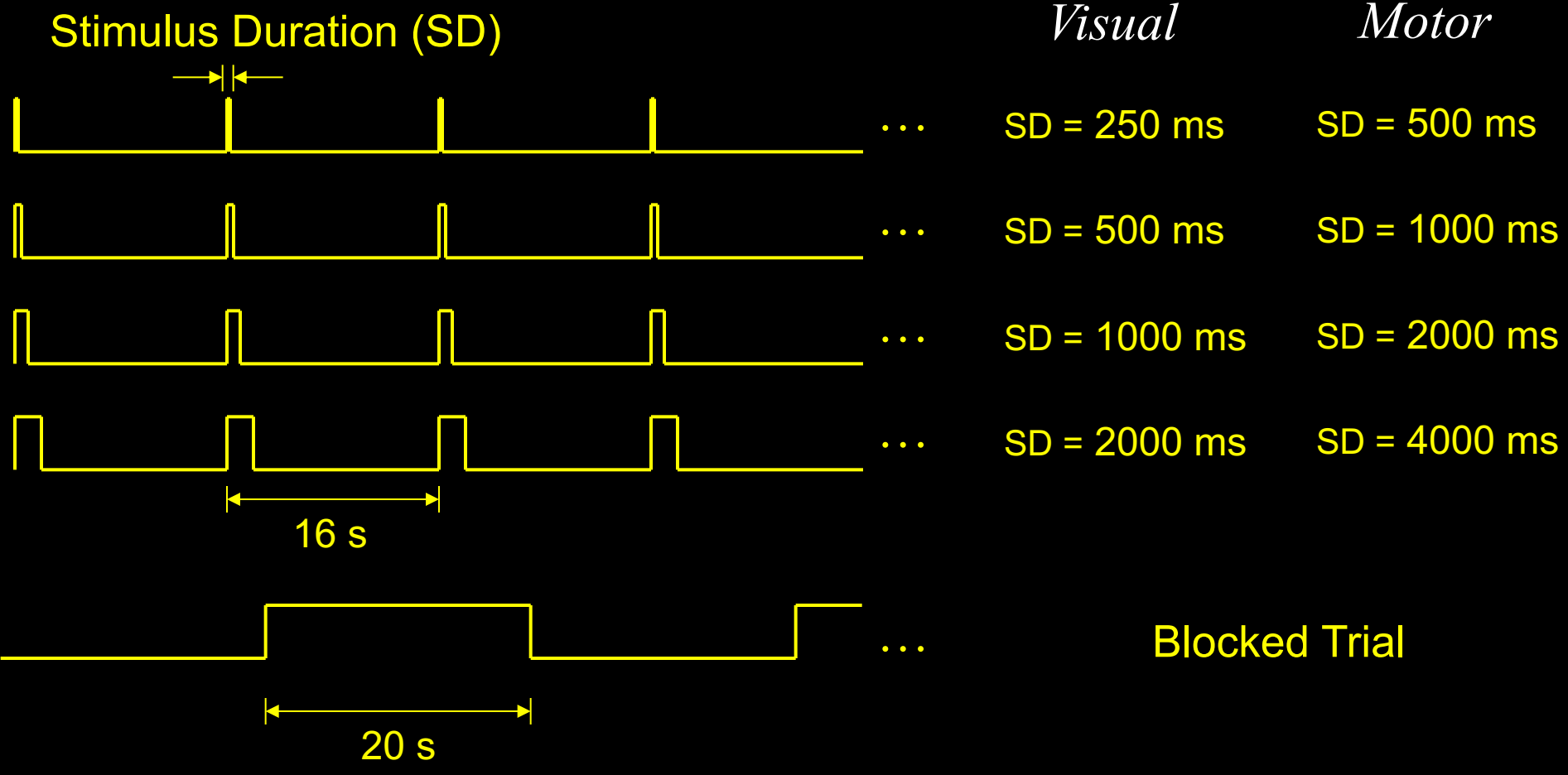
# Auditory Cortex







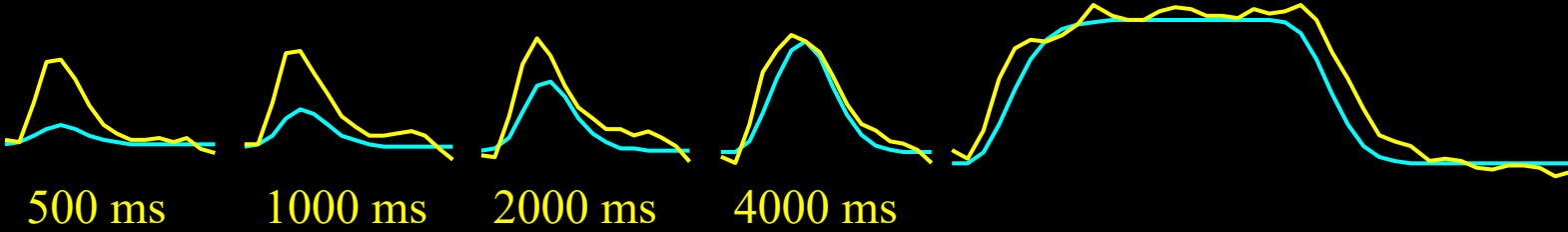
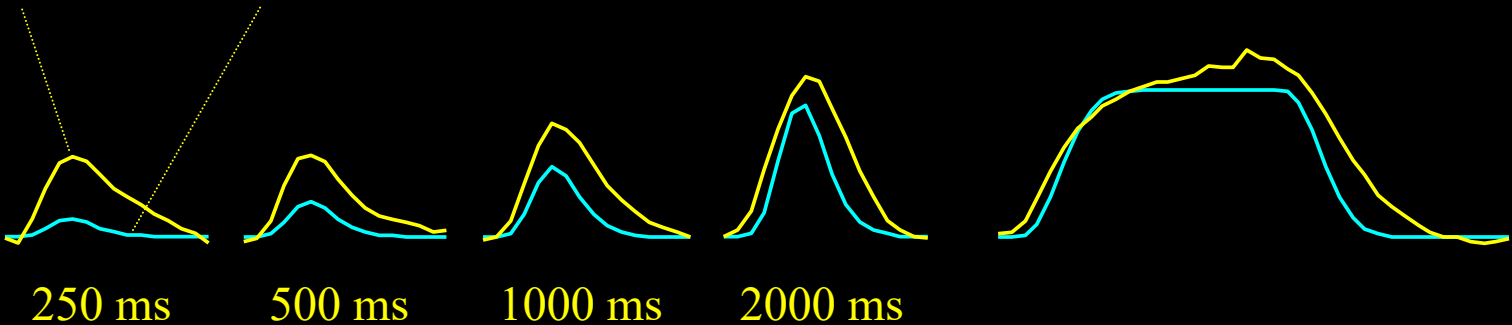
# Methods



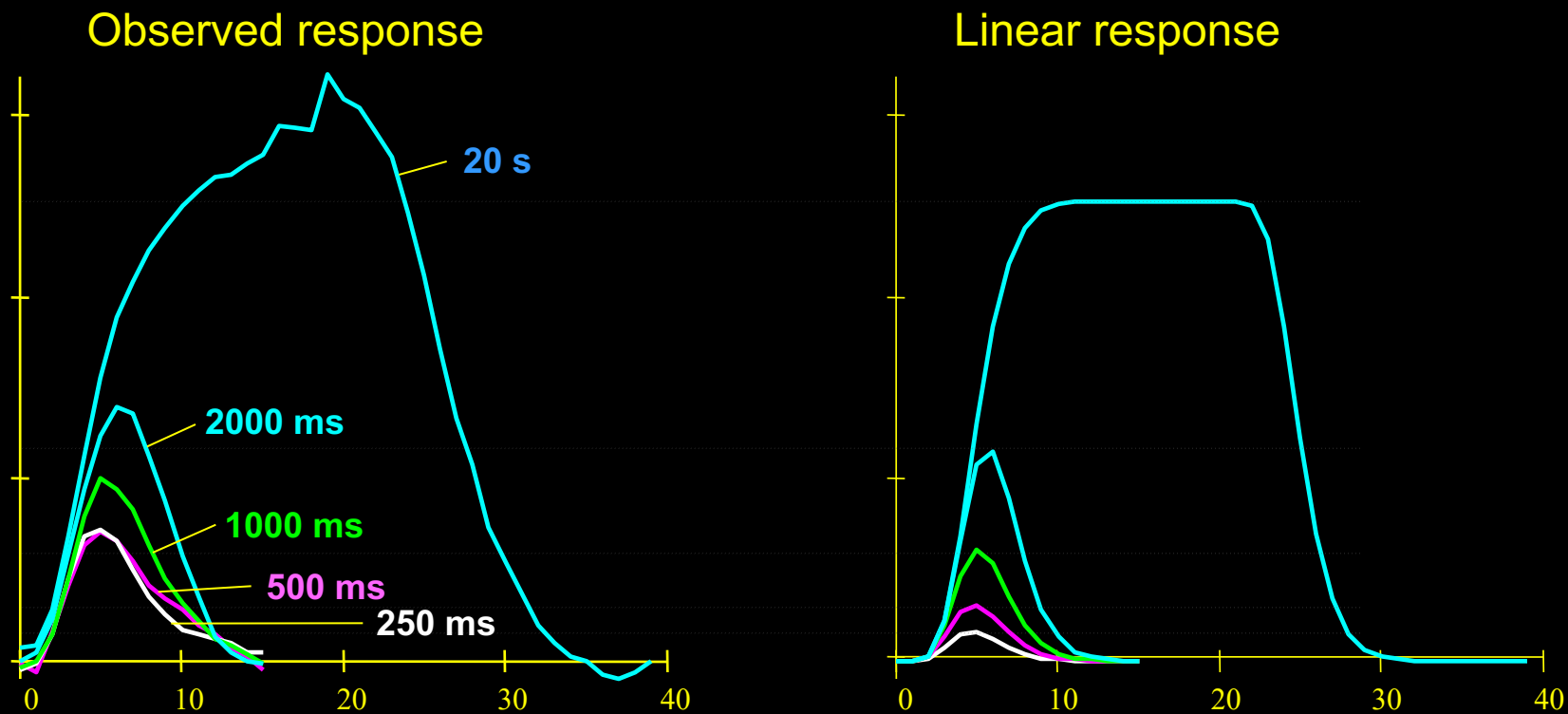
# Observed Responses

*measured*

*ideal (linear)*



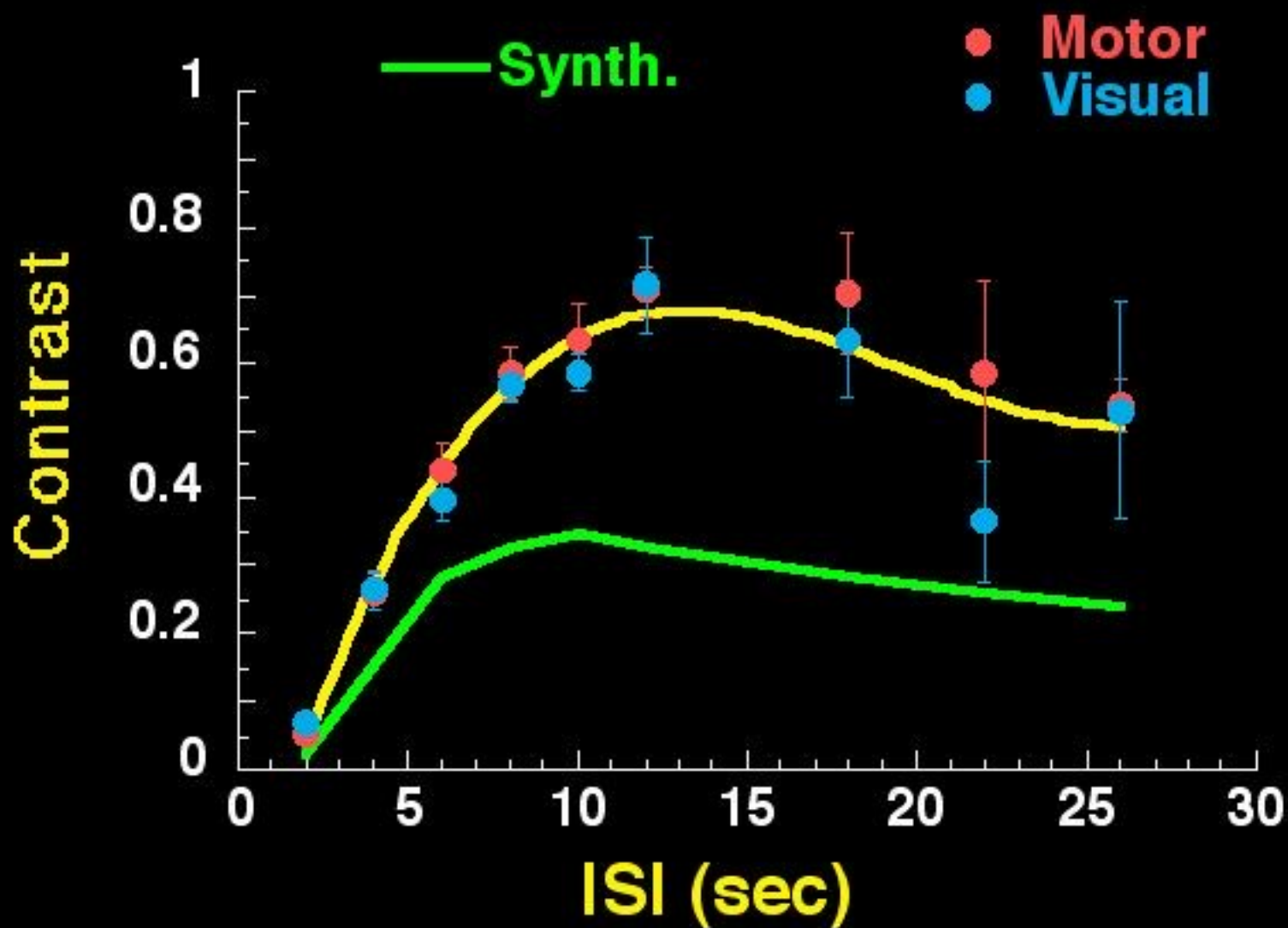
# BOLD response is nonlinear



*Short duration stimuli produce larger responses than expected*



# Functional Contrast



( Block design = 1 )

# Contrast to Noise Images

( ISI, SD )

20, 20

12, 2

10, 2

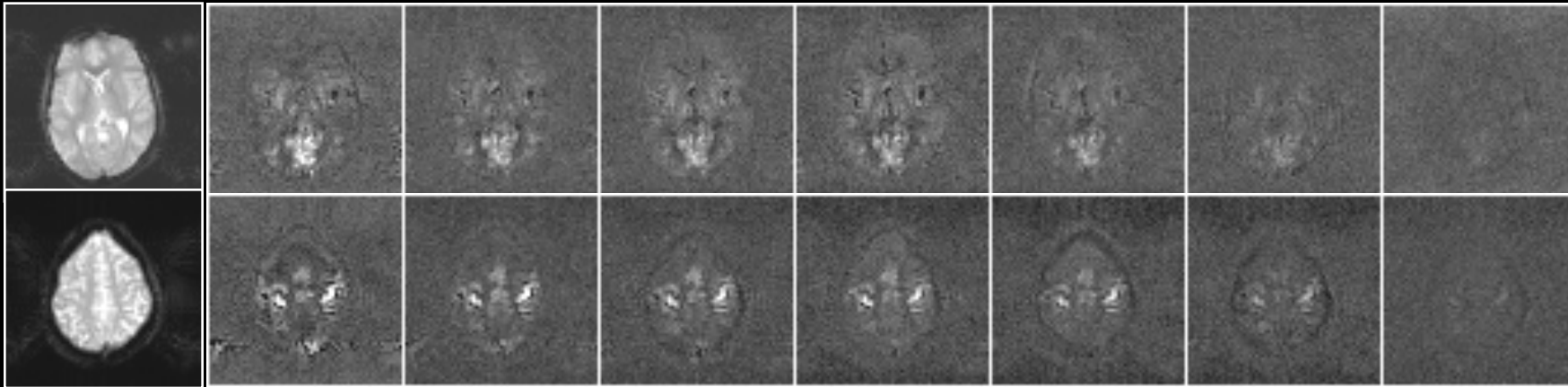
8, 2

6, 2

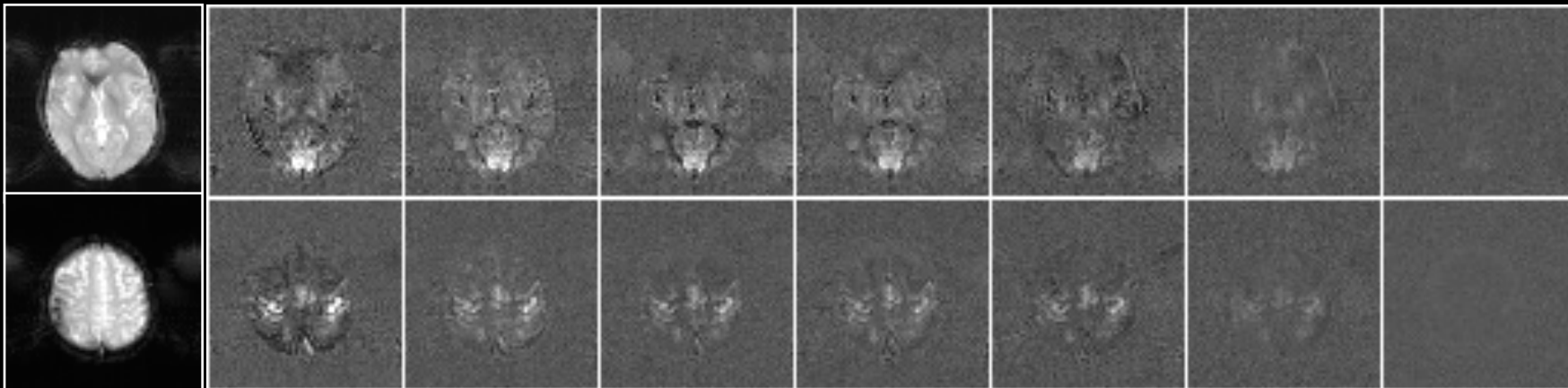
4, 2

2, 2

S1



S2



## Source of the Nonlinearity

Neuronal

Hemodynamic

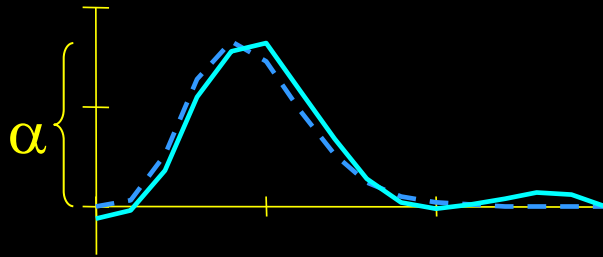
*Miller et al. 1998* – Flow is linear, BOLD is nonlinear

*Friston et al. 2000* – hemodynamics can explain nonlinearity

*If nonlinearity is hemodynamic in origin, a measure of this nonlinearity will reflect any spatial variation of the vasculature*

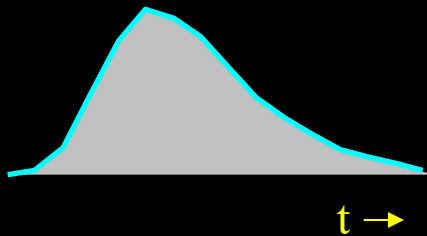
## Compute nonlinearity (*for each voxel*)

- Amplitude of Response



*Fit ideal (linear) to response*

- Area under response / Stimulus Duration



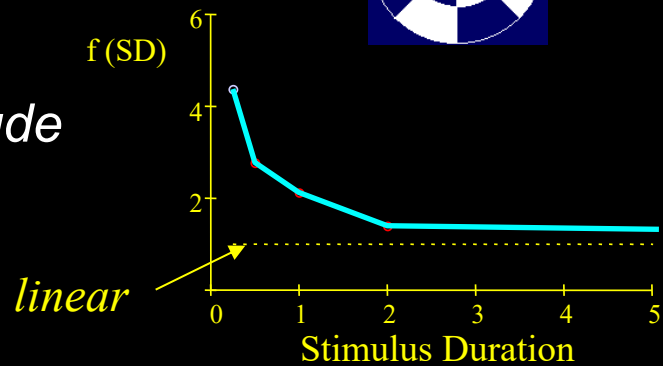
*Output Area / Input Area*

# Nonlinearity

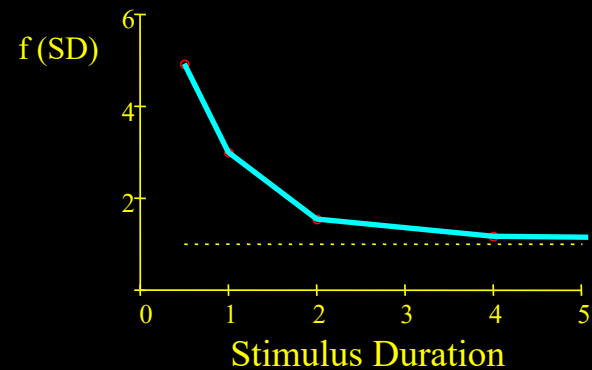
Visual



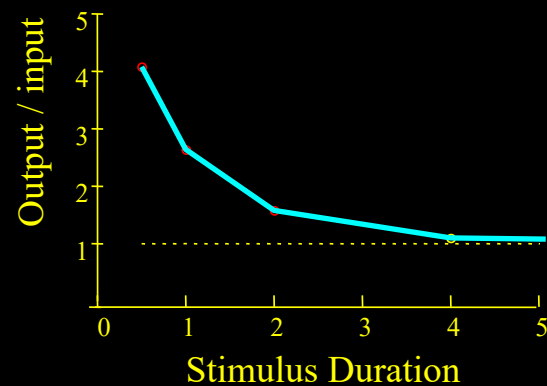
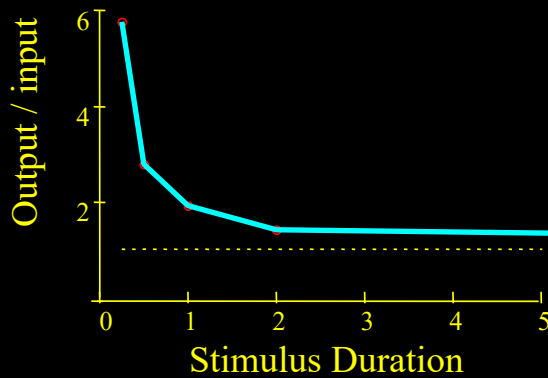
Magnitude



Motor

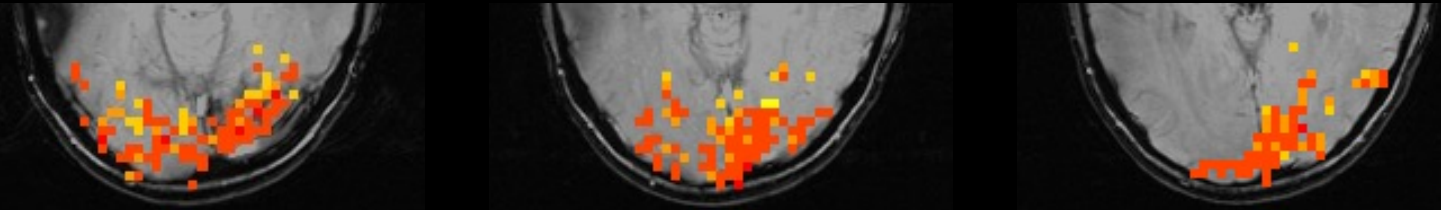


Area

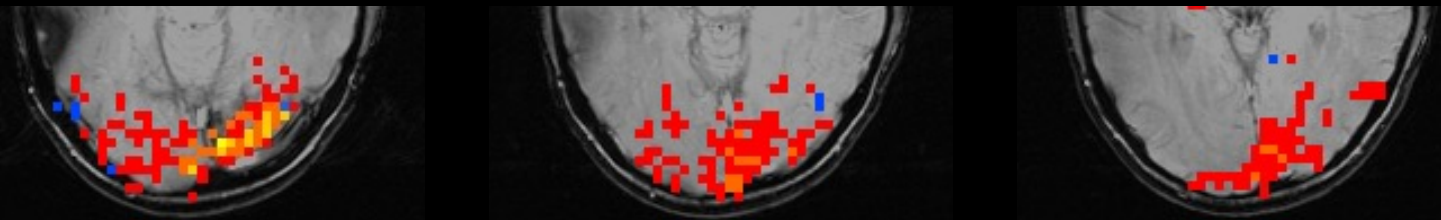


# Results — visual task

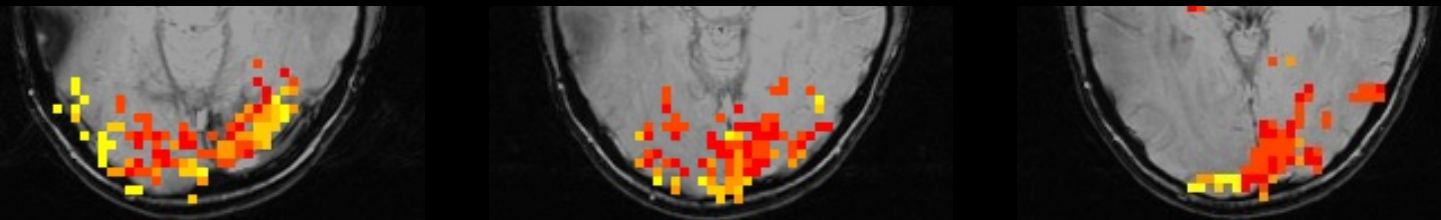
Nonlinearity



Magnitude

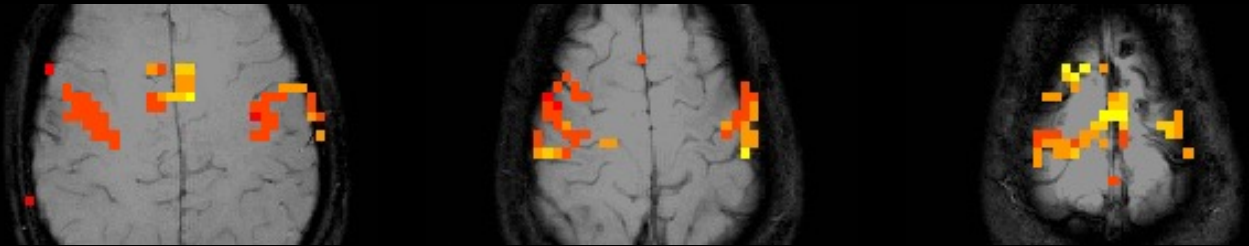


Latency

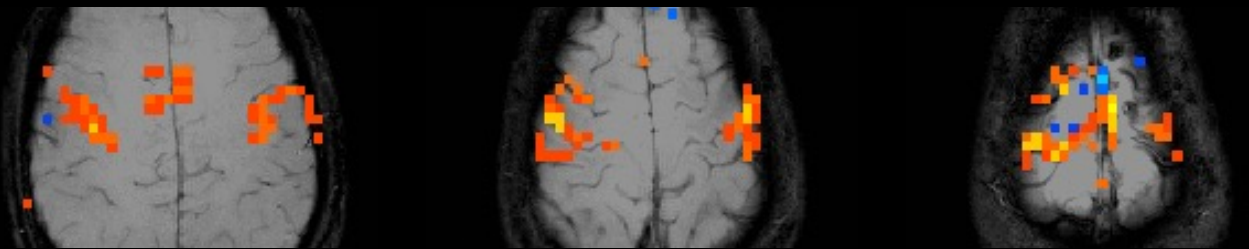


# Results — motor task

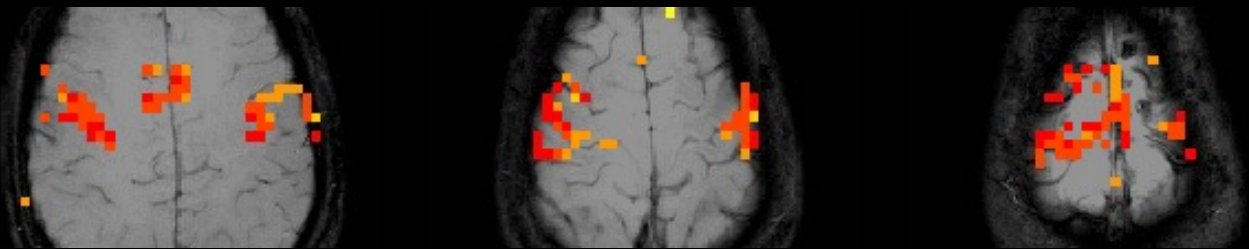
Nonlinearity



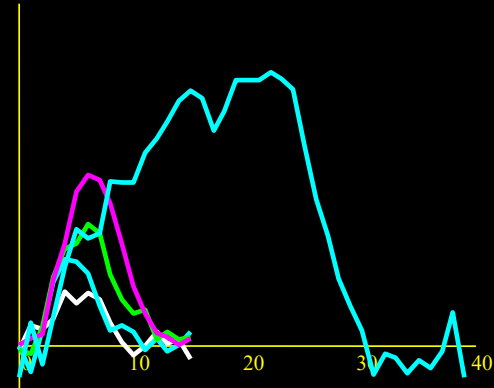
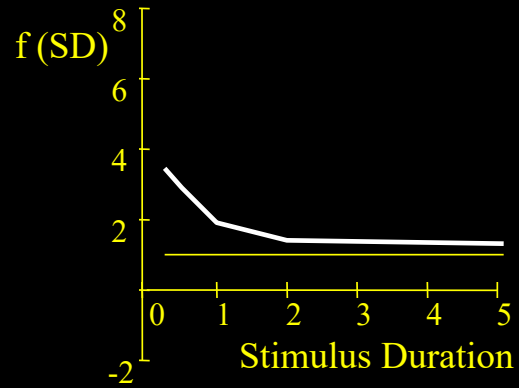
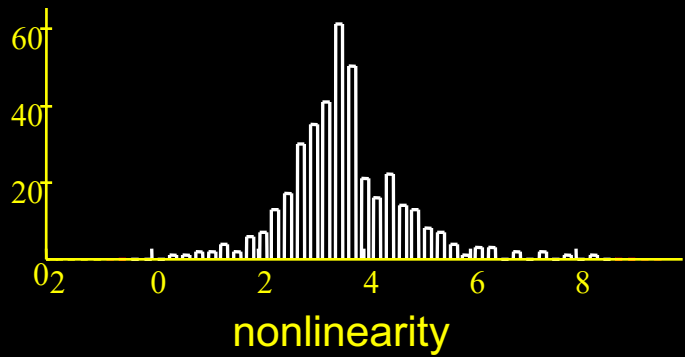
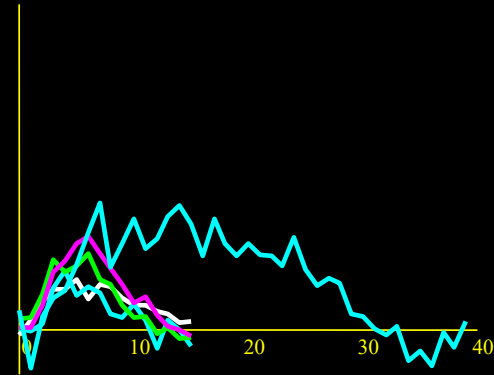
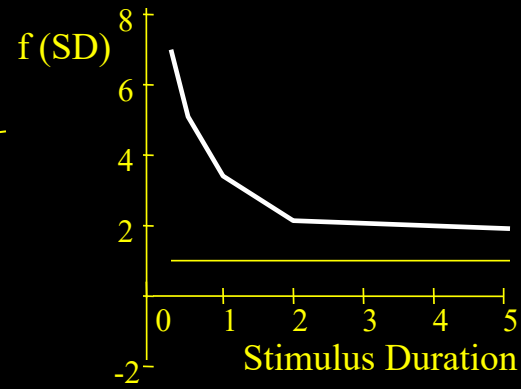
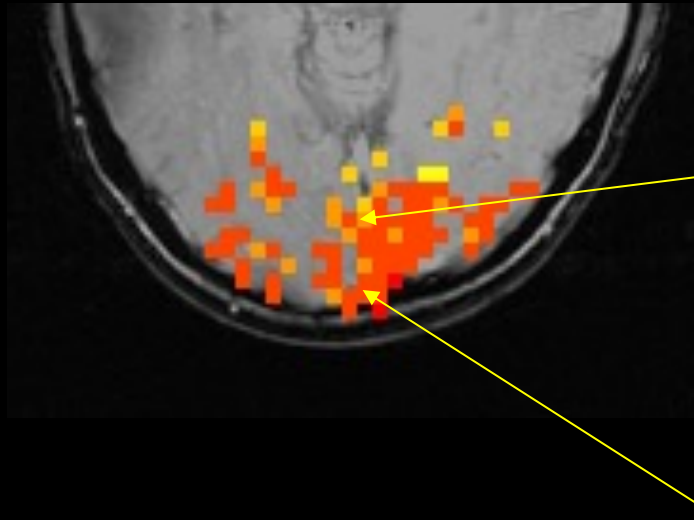
Magnitude



Latency

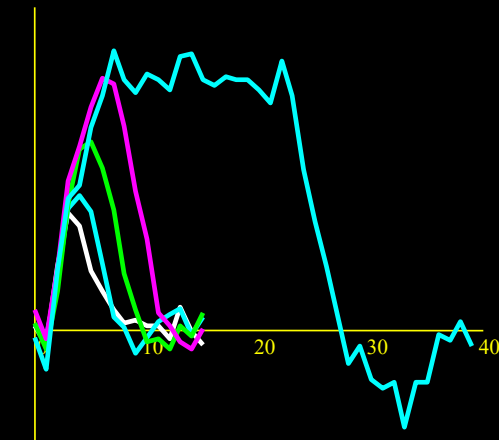
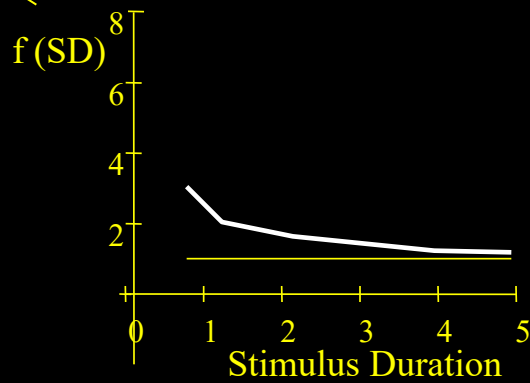
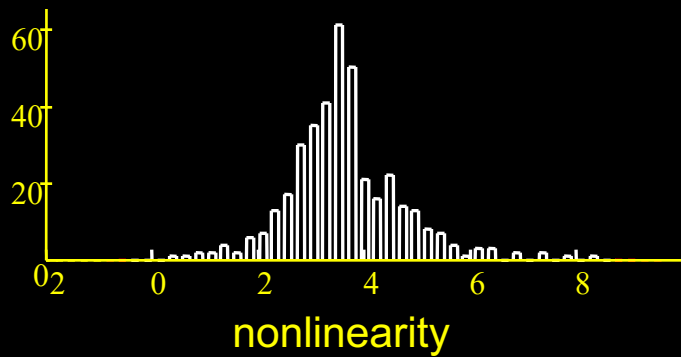
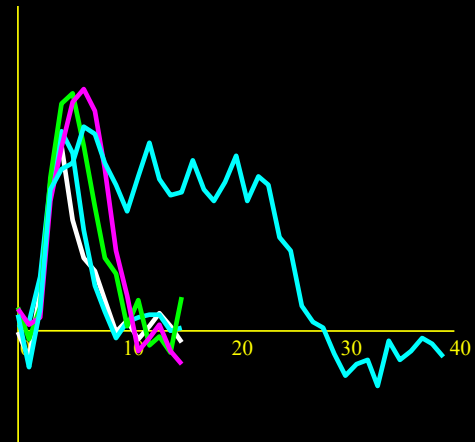
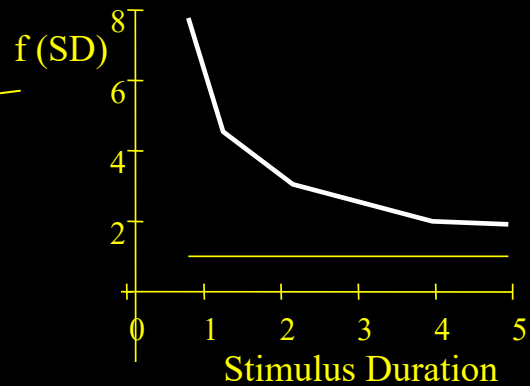
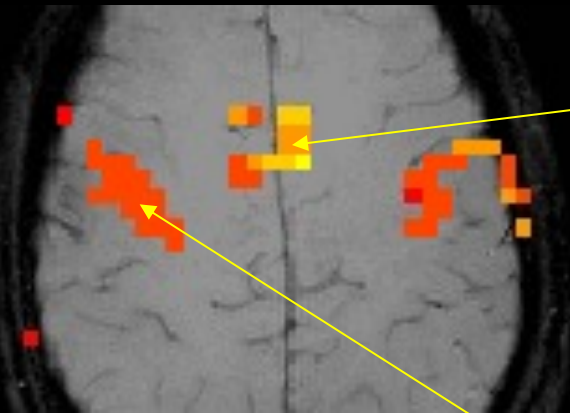


# Results — visual task



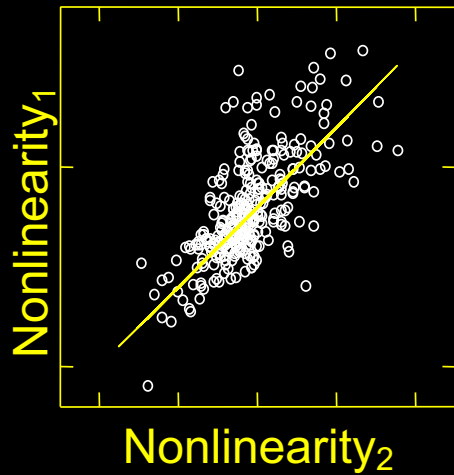


# Results — motor task

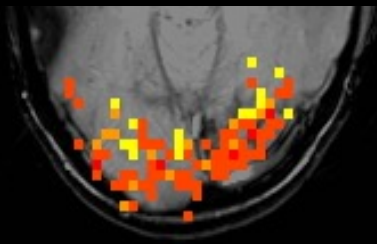
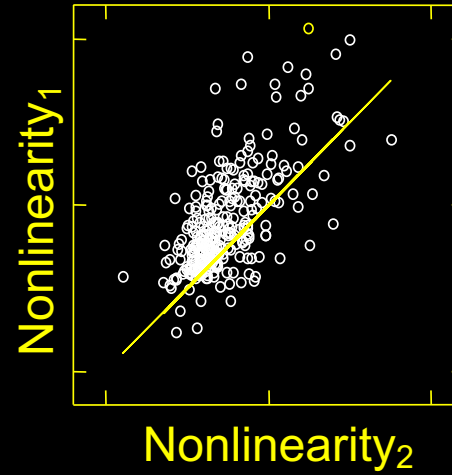


# Reproducibility

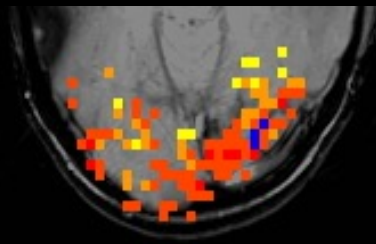
*Visual task*



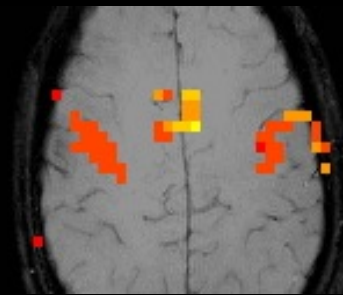
*Motor task*



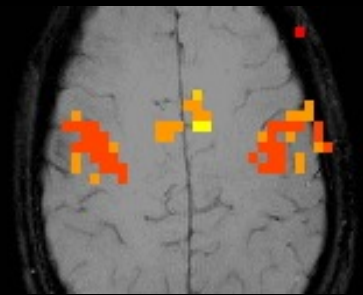
Experiment 1



Experiment 2

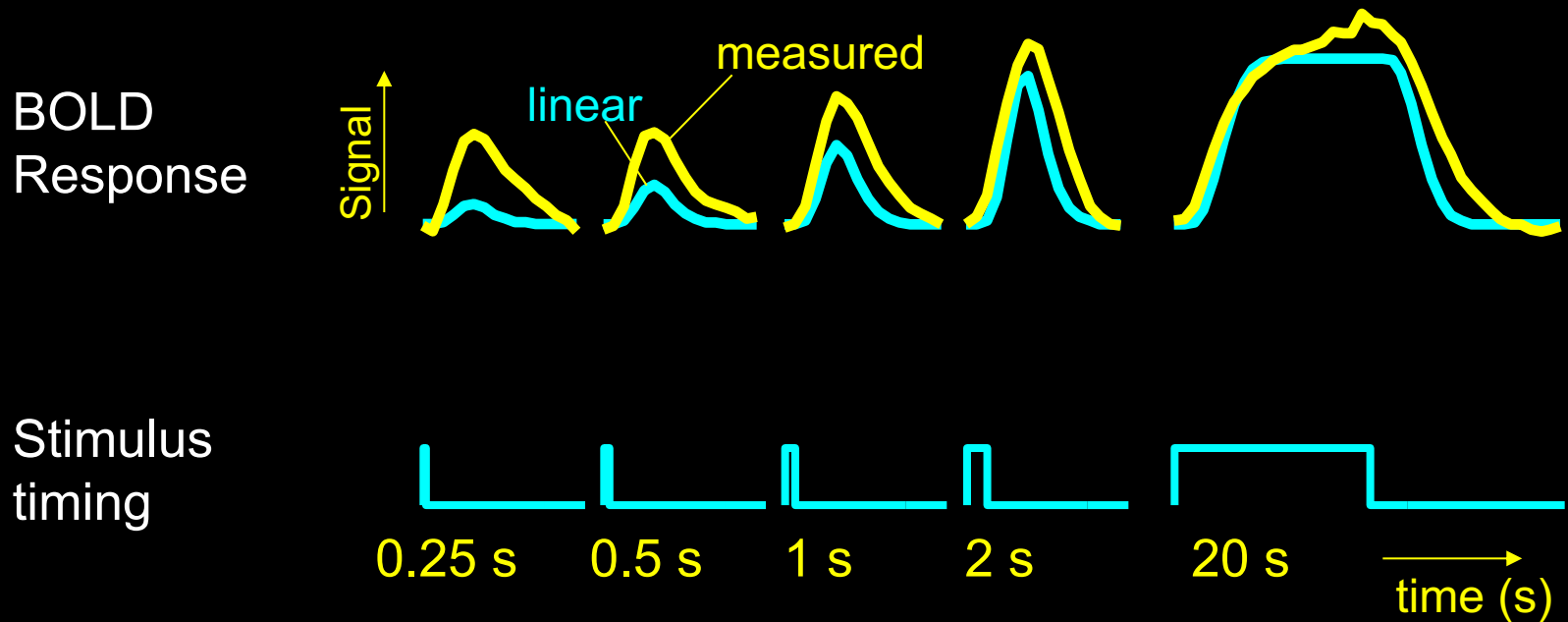


Experiment 1



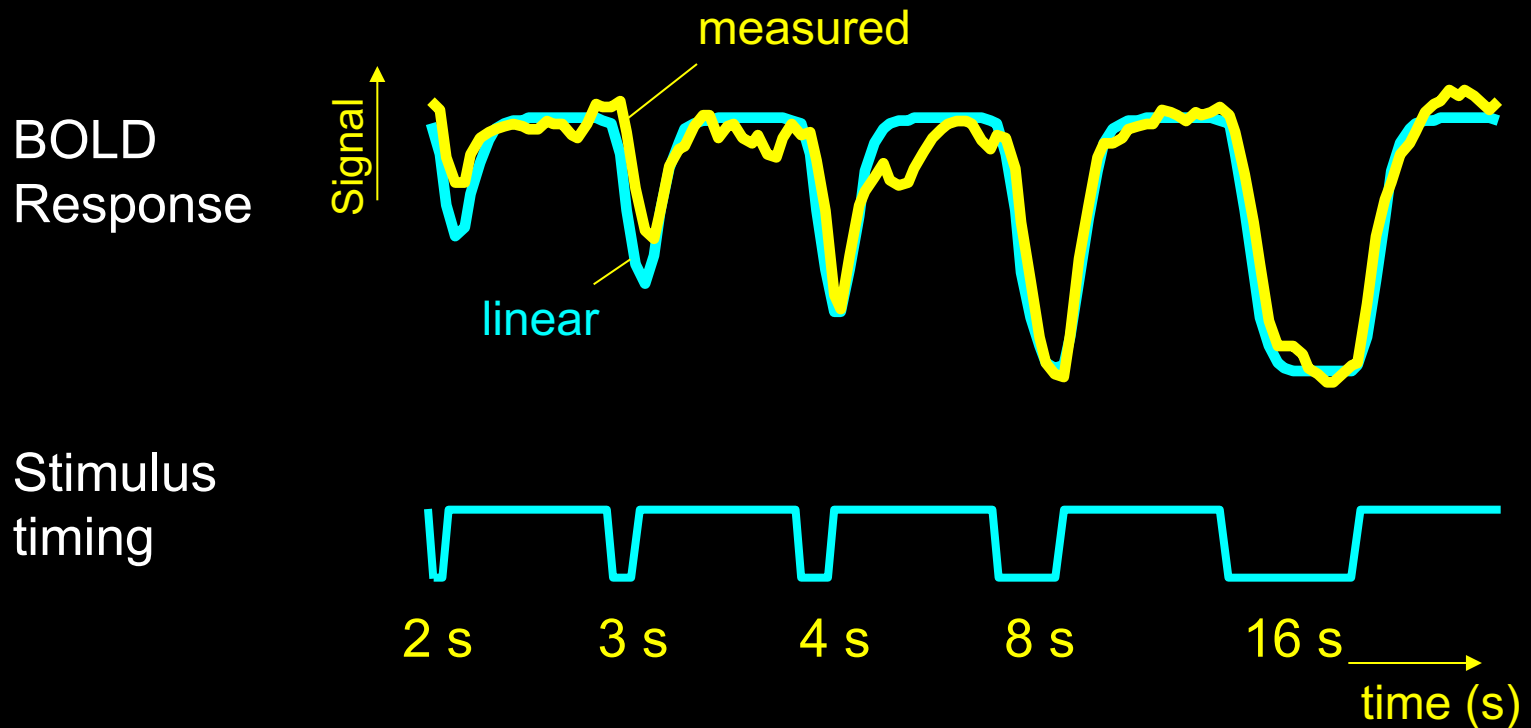
Experiment 2

# Different stimulus “ON” periods



*Brief stimuli produce larger responses than expected*

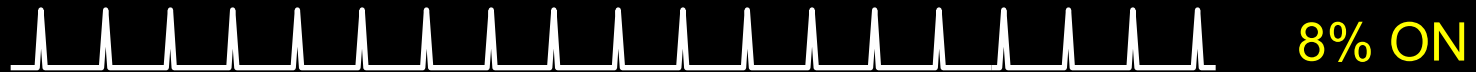
# Different stimulus “ON” periods



*Brief stimulus OFF periods produce smaller decreases than expected*

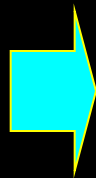
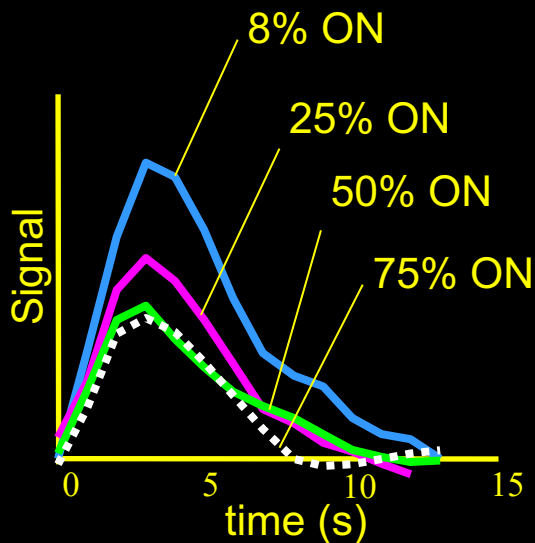
# Varying “ON” and “OFF” periods

- *Rapid event-related design with varying ISI*

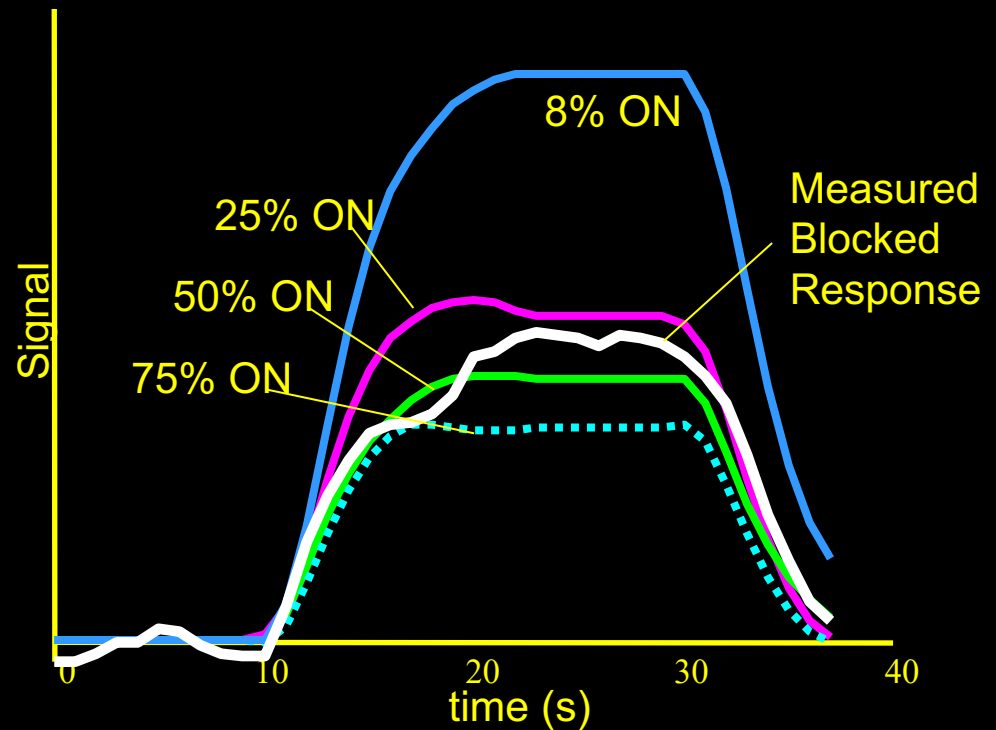


# Varying “ON” and “OFF” periods

*Estimated  
Impulse Response*



*Predicted Responses  
to 20 s stimulation*

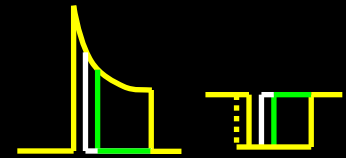
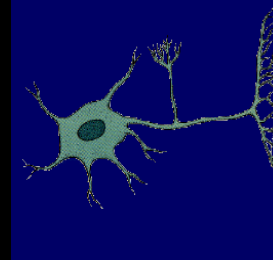


# Conclusions

- For brief stimulus “ON” periods, signal increases are larger than expected. These nonlinearities show considerable yet reproducible spatial heterogeneity.
- For brief stimulus “OFF” periods, signal decreases are smaller than expected
- For varying “ON” and “OFF” periods, deconvolved impulse response depends on fraction of time in “ON” state.

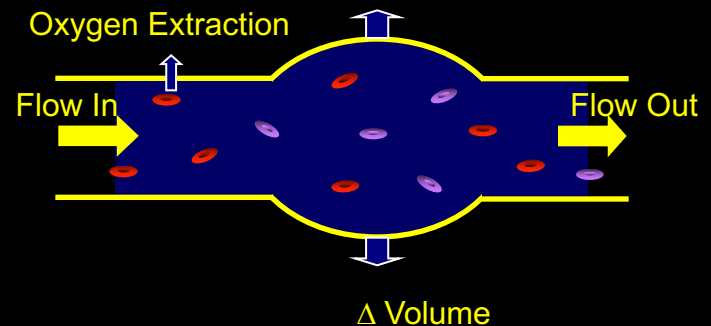
# Sources of this Nonlinearity

- Neuronal

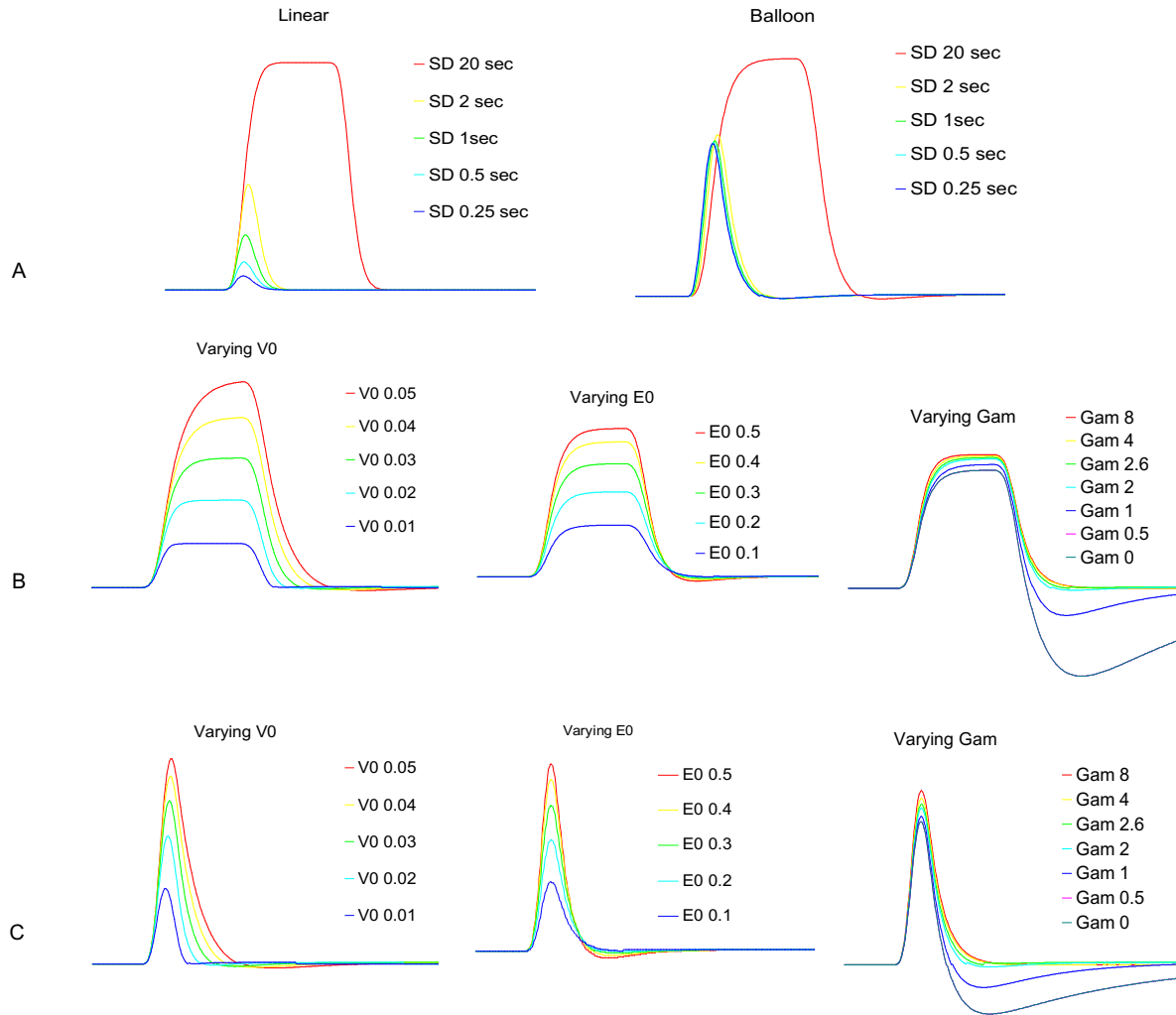


- Hemodynamic

- Oxygen extraction
- Blood volume dynamics







**BOLD Curves**

- A Linear and Nonlinear “Balloon” BOLD Curves for Varying SD 20,2,1,0.5, 0.25 sec
- B Balloon Curves, SD = 20 sec: One parameter is varied at a time. When not varied they are set equal to  $V_0 = 0.03$ ,  $E_0 = 0.3$ , and  $\text{Gam} = 2.6$
- C Balloon Curves, SD = 2 sec: One parameter is varied at a time. When not varied they are set equal to  $V_0 = 0.03$ ,  $E_0 = 0.3$ , and  $\text{Gam} = 2.6$

# Balloon Model Parameters

For a given flow of blood into the venous compartment, the three Balloon parameters which control the hemodynamic contribution to the BOLD signal are thought to be  $E_0$ ,  $V_0$ , and  $\text{Gam}$ .

$E_0$  represents the fraction of total hemoglobin not bound to  $O_2$ ;

$v(t)$  is the fraction of voxel volume filled with blood during the active state normalized to that at rest,  $V_0$ ;

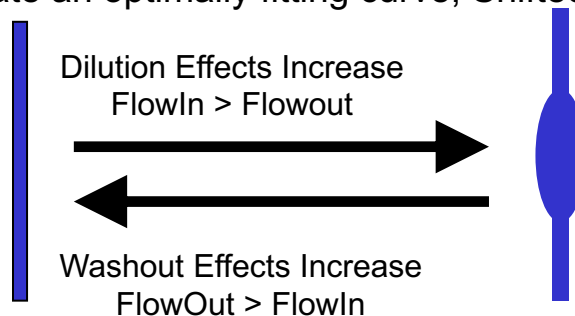
$\tau_0$  is the mean venous transit time of blood in the venous compartment and equals  $V_0 / \text{FlowOut}(0)$ ;

$\text{Gam}$  is the exponent defining the relationship between venous outflow and fractional blood volume;

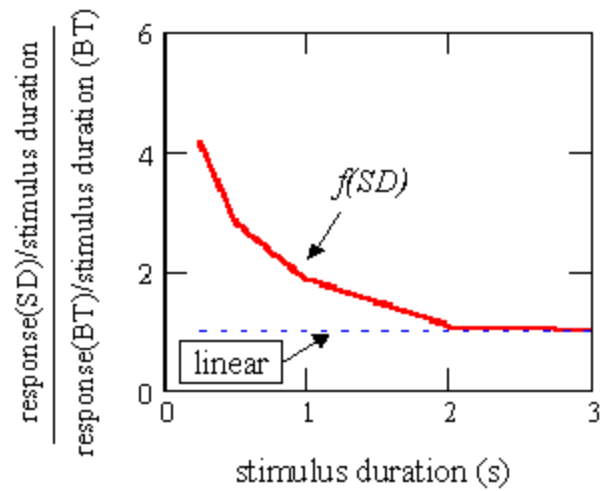
$q(t)$  is the total voxel content of dHB during the active state normalized to that at rest;

$\text{viscos}$  is a viscosity term that varies between viscup, during balloon inflation, and viscdown, during balloon deflation.

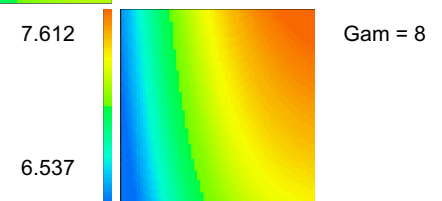
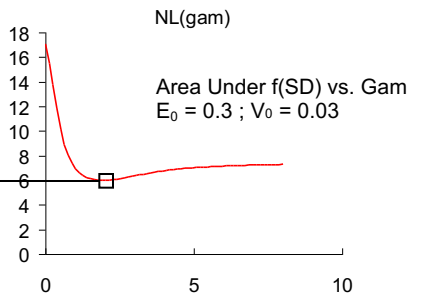
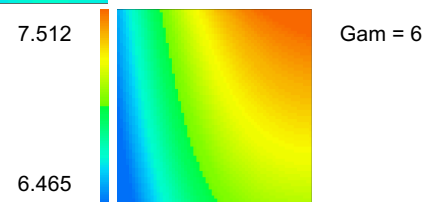
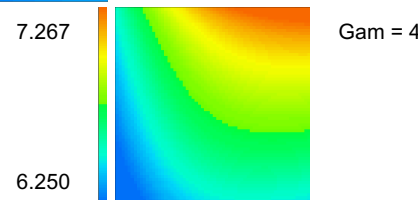
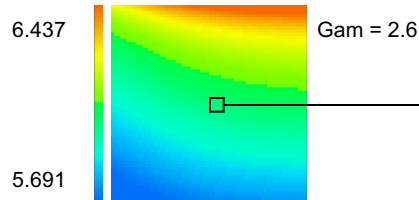
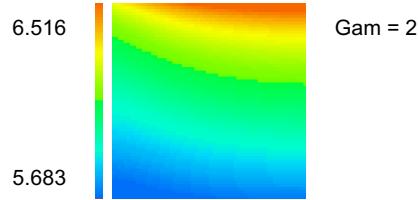
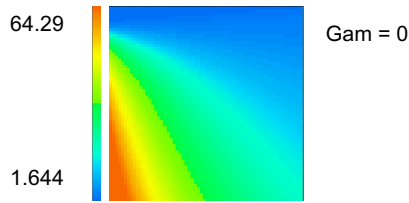
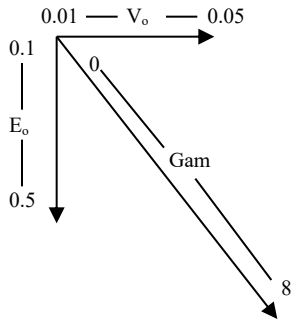
On a voxelwise basis, the stimulus waveform was smoothed ( $\text{WAVrisetime}$ ), scaled ( $\text{FLINamp}$ ), and phase shifted ( $\text{FLINdelay}$ ) in order to generate an optimally fitting curve,  $\text{ShiftedFlowIn}(t)$ , representing blood flow into the venous compartment.



Nonlinearity = Area Under  $f(SD)$  greater than 1



# Balloon Model Nonlinearity Maps



$E_0$ , dHb fraction;  
 $V_0$ , blood volume fraction at rest;  
 $\text{Gam}$ , steady state venous outflow-volume relationship  
 $\tau_0$ , the mean venous transit time of blood;

## Physiological Parameter Contribution to NL

|                          |                                       |
|--------------------------|---------------------------------------|
| $\text{Gam} \ll 2$       | $\text{Gam} \gg (\tau_0 > E_0)$       |
| $2 \ll \text{Gam} \ll 4$ | $\tau_0 \gg (E_0 > \text{Gam})$       |
| $\text{Gam} \approx 4$   | $(\tau_0 \approx E_0) \gg \text{Gam}$ |
| $4 \ll \text{Gam} < 8$   | $E_0 \gg (\tau_0 > \text{Gam})$       |

## Main NL dependence

$\text{Gam} \ll 2$

$v(t)$  and  $\text{FlowOut}(t)$

$\tau_0$   
 $E_0$

NL Range  $\gg 1$

|                  |                   |
|------------------|-------------------|
| <u>Larger NL</u> | <u>Smaller NL</u> |
| Shorter          | Longer            |
| Larger           | Smaller           |

$\text{Gam} \geq 2$

$\tau_0$  and  $E_0$

$\tau_0$   
 $E_0$

NL Range  $\approx 1$

|                  |                   |
|------------------|-------------------|
| <u>Larger NL</u> | <u>Smaller NL</u> |
| Longer           | Shorter           |
| Smaller          | Larger            |

For TE = 30ms

$k_1$       1.5T  
                $5.2 * E_0$   
 $k_2$        $1.43 * E_0$   
 $k_3$       0.43

3.0T  
 $10.4 * E_0$   
 $0.5 * E_0$   
 -0.5

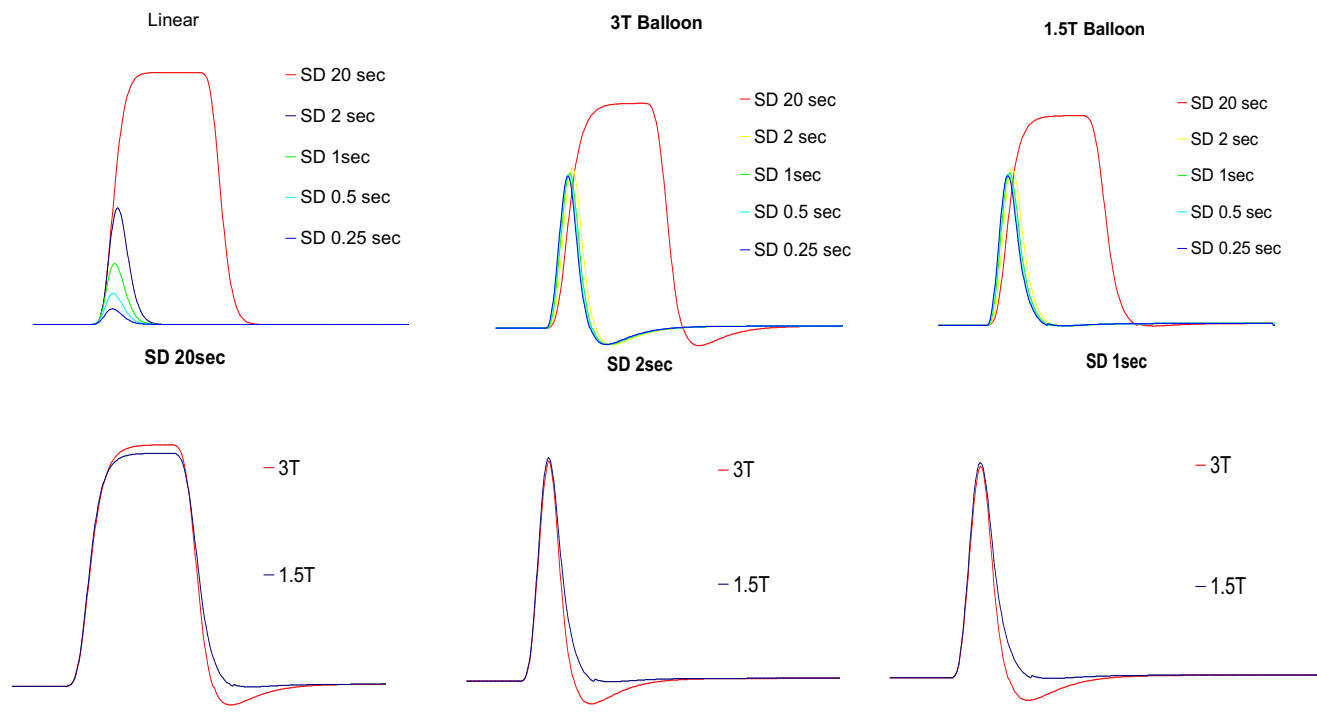


Figure 2:  
 Balloon Curves at different Tesla, SD = 20 sec.  
 $V_0 = 0.03$ ,  $E_0 = 0.3$ , and  $\text{Gam} = 2.6$

## Conclusions

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When varied independently,  $E_0$ ,  $V_0$ , and  $\Gamma_{\text{am}}$  each affect the BOLD signal in different ways. The interaction of these parameters produces BOLD curves that are nonlinear when compared to the linear results using the same SD.

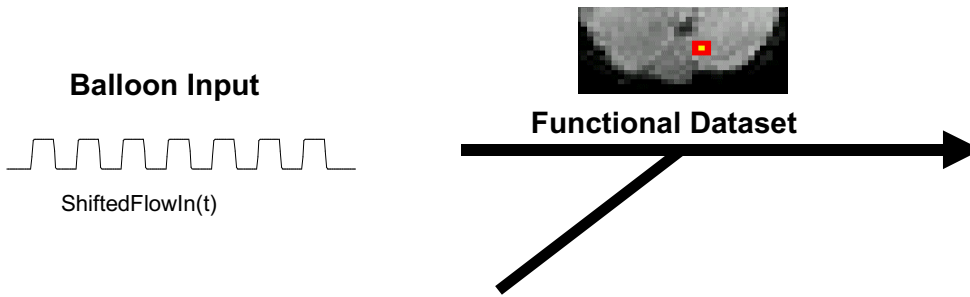
For  $\Gamma_{\text{am}}$  values between 0 and 2, in which venous outflow is not laminar, small increases in  $\Gamma_{\text{am}}$  reduce nonlinearity (NL).

Nonlinearity is a function of several parameters, whose relative contributions to NL are determined by the value of each parameter.

For  $\Gamma_{\text{am}}$  values between 2.1 to 6.4 and with other parameters in physiological range, NL values ranged from 6.01 to 7.53.

By limiting the NL range to the range of NL obtained experimentally ( NL between 5 and 10), the balloon model can be further constrained in our attempt to extract physiologic information from the BOLD response in humans. Further analysis is necessary to determine how varying the viscoelasticity of the venous compartment affects NL.

# Overview



## Balloon Model Equations

$$\frac{\Delta S}{S} = V_0 [(k1 + k2)(1 - q(t)) - (k2 + k3)(1 - v(t))]$$

$$\text{Exfrac}(t) = 1 - (1 - E_0) \frac{1}{\text{ShiftedFlowIn}(t)}$$

$$\text{CMRO}_2(t) = \text{ShiftedFlowIn}(t) * \frac{\text{Exfrac}(t)}{E_0}$$

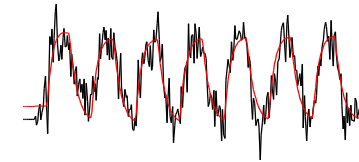
$$\text{FlowOut}(t) = v(t)^{\text{Gam}}; \quad \frac{df}{dv} = \text{Gam}(v(t))^{(\text{Gam}-1)}$$

$$\tau_0 = \frac{V_0}{\text{FlowOut}(0)}$$

$$q(t) = \frac{Q(t)}{Q_0}; \quad \frac{dq}{dt} = \frac{1}{\tau_0} \left[ \text{ShiftedFlowIn}(t) \frac{\text{Exfrac}(t)}{E_0} - \text{FlowOut}(t) \frac{q(t)}{v(t)} \right]$$

$$v(t) = \frac{V(t)}{V_0}; \quad \frac{dv}{dt} = \frac{1}{\tau_0} \left[ \frac{\text{ShiftedFlowIn}(t) - \text{FlowOut}(t)}{1 + 0.5 \left( \frac{dt}{\tau_0} \right) \left( \frac{df}{dv} \right) + \left( \frac{\text{viscos}}{\sqrt{v(t)}} \right)} \right]$$

## Optimized Balloon Output



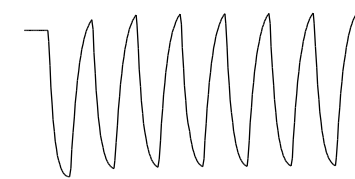
Balloon Fit to BOLD Signal



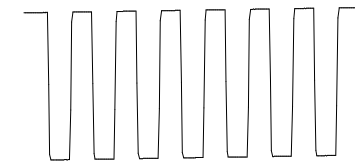
FlowOut(t)



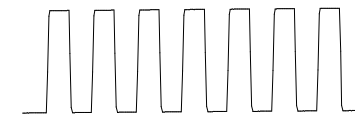
v(t)



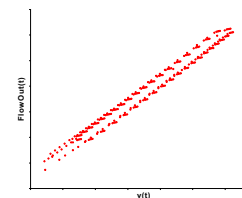
q(t)



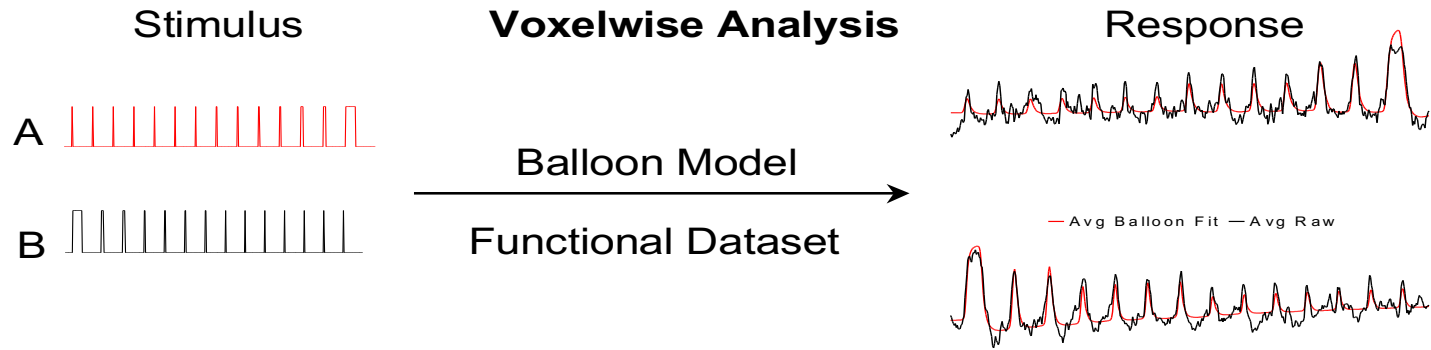
Exfrac(t)



CMRO<sub>2</sub>



FlowOut(t) versus v(t)



Relevant Physiologic Range

$E_0$                       0.2 to 0.4

$V_0$                       0.02 to 0.05

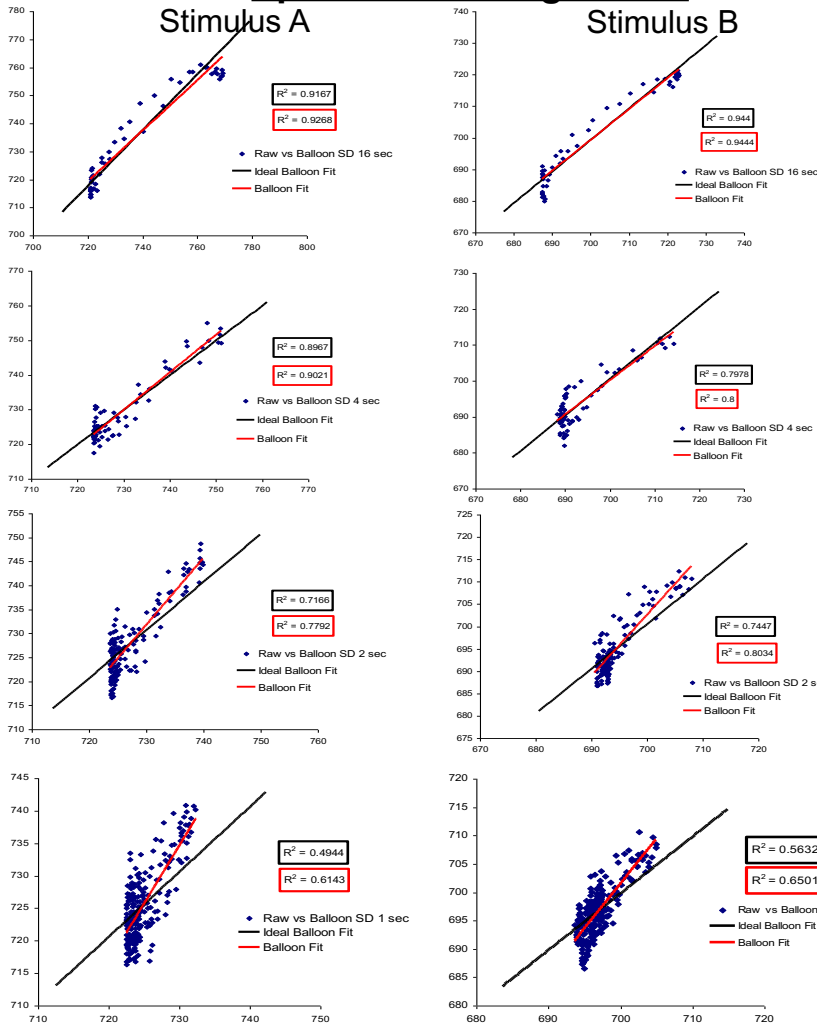
$\text{Gam}$                       2.1 to 6.4

## Balloon Model Parameter Estimation

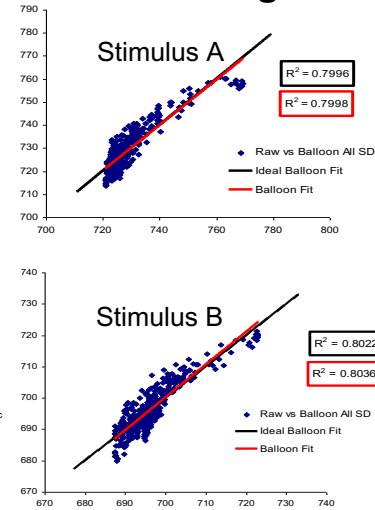
|              | A1      | A2      | Mean    | StdDev | %StDev/Mean | B1      | B2      | Mean    | StdDev | %StDev/Mean |
|--------------|---------|---------|---------|--------|-------------|---------|---------|---------|--------|-------------|
| constant     | 726.422 | 719.873 | 723.148 | 4.631  | 0.640       | 687.650 | 695.451 | 691.551 | 5.516  | 0.798       |
| linear       | -0.008  | 0.029   | 0.011   | 0.026  | 241.779     | 0.023   | 0.005   | 0.014   | 0.013  | 94.457      |
| FLINamp      | 0.598   | 0.491   | 0.545   | 0.076  | 13.938      | 0.582   | 0.603   | 0.592   | 0.015  | 2.498       |
| FLINdelay    | -0.794  | -0.808  | -0.801  | 0.010  | -1.227      | 0.662   | 0.545   | 0.604   | 0.083  | 13.748      |
| $V_0$        | 0.051   | 0.049   | 0.050   | 0.002  | 3.825       | 0.034   | 0.041   | 0.037   | 0.004  | 12.007      |
| $E_0$        | 0.330   | 0.295   | 0.312   | 0.025  | 7.982       | 0.436   | 0.393   | 0.415   | 0.030  | 7.288       |
| $\text{Gam}$ | 4.151   | 3.723   | 3.937   | 0.303  | 7.687       | 3.742   | 3.495   | 3.618   | 0.175  | 4.830       |
| WAVrisetime  | 2.572   | 2.788   | 2.680   | 0.153  | 5.706       | 2.431   | 2.625   | 2.528   | 0.138  | 5.445       |
| viscup       | 3.780   | 3.206   | 3.493   | 0.406  | 11.620      | 8.529   | 7.115   | 7.822   | 1.000  | 12.782      |
| viscdwn      | 8.870   | 11.086  | 9.978   | 1.567  | 15.704      | 9.945   | 10.250  | 10.098  | 0.215  | 2.133       |



## Epochs in Averaged Run



## Entire Averaged Run



Magnitudes of Averaged Raw Data

Magnitudes of Averaged Balloon Model Fit

Raw Experimental Data versus the Optimal Balloon Model Fit

The magnitudes for different stimuli (A and B), averaged across two runs, are plotted for epochs (16, 4, 2, 1 sec) within an averaged run and for all epochs in the averaged run).

## Conclusions

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Balloon model hemodynamics do not fully account for human BOLD signal NL.

Within a run for a given stimulus, epochs of longer stimulus duration are better characterized by the Balloon model than shorter stimulus durations.

As epoch durations become briefer, the Balloon model fits become more linear relative to experimental data.